Entropy analysis for fusion kinetic plasmas: turbulence and energetic particle instability

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2024/ 9/ 19

PPPL theory seminar

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CONTENTS

Hanyang University (HYU)

- Hanyang
= Old name of Seoul

- 15K students, 52 departments

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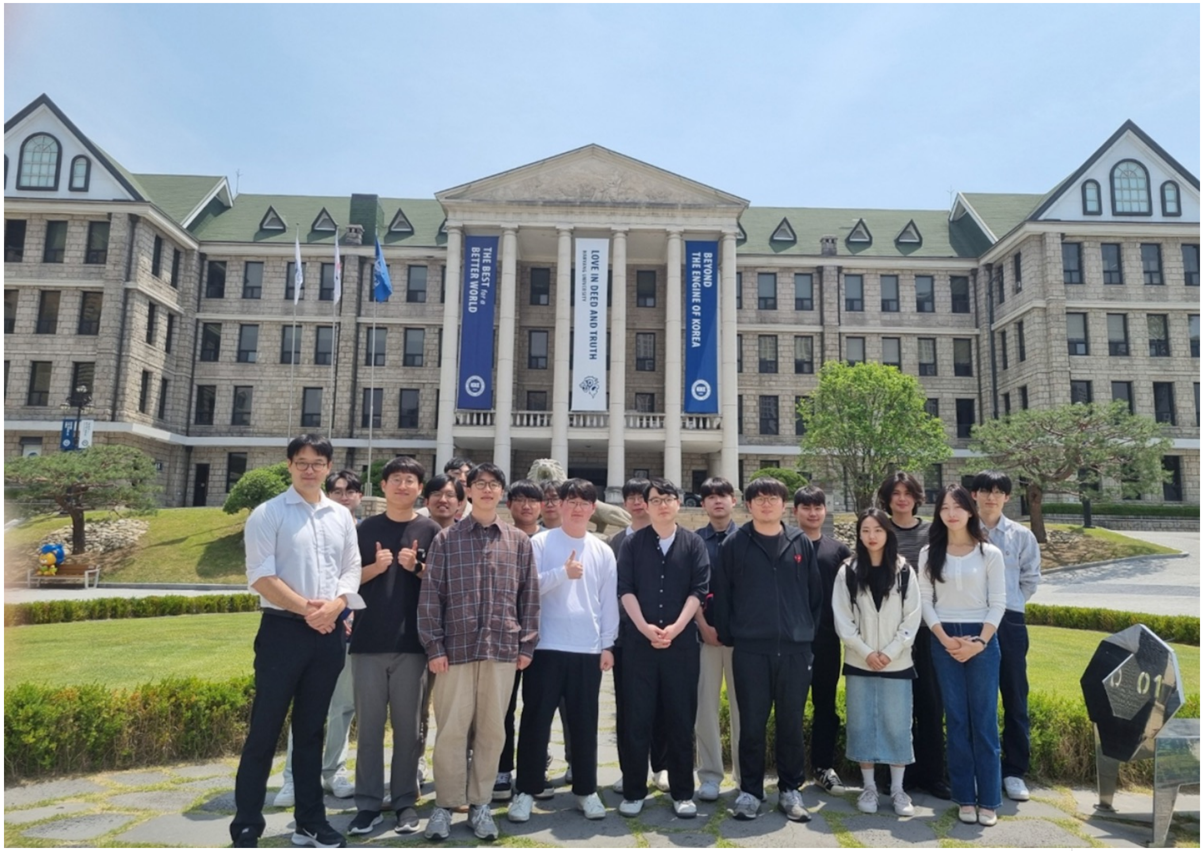
Nuclear Fusion and Plasma Computation Group

- Established in 2018
- Nuclear Engineering Dep.
- 9 graduate students, 4 undergraduate students
- Research on Nuclear fusion theory and computation (Heating, Transport, MHD, ..), and semiconductor plasma application
- Collaboration with KFE, GA, SNU, KAIST, UNIST, MIT, PPPL, Columbia U, SEMES.



Ongoing Project 1 – ECH V&V and ML

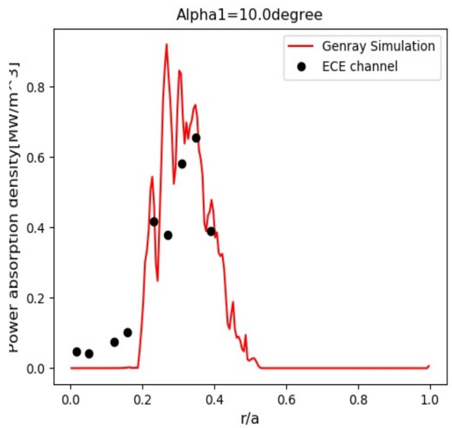
- KSTAR ECH vs. NBI heating

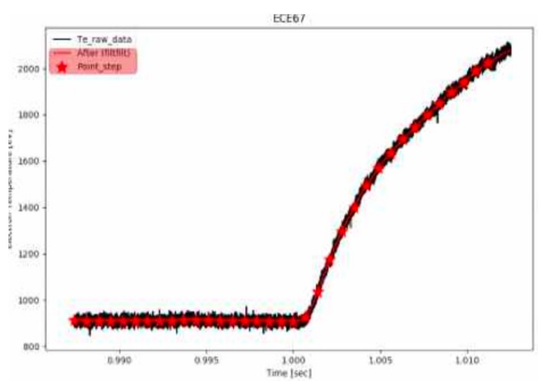
- Twice larger W_{mhd} increase for NBI

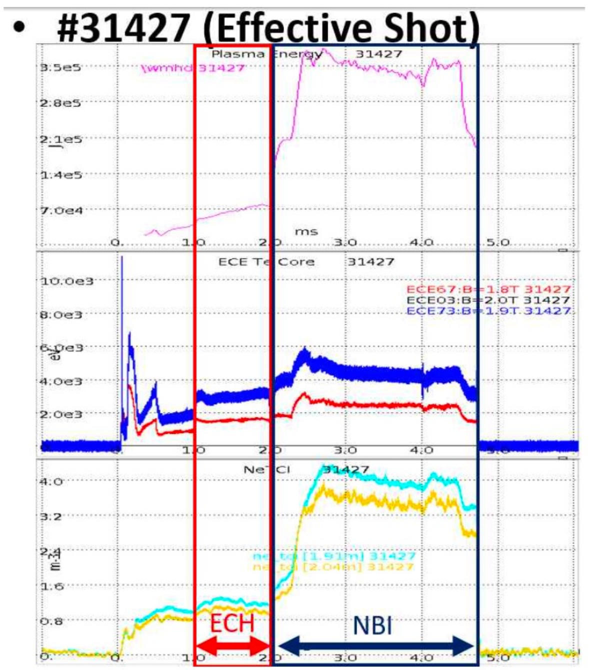
- ECH Power absorption verification by GENRAY and ECE temperature change

→ Power absorption could be fine

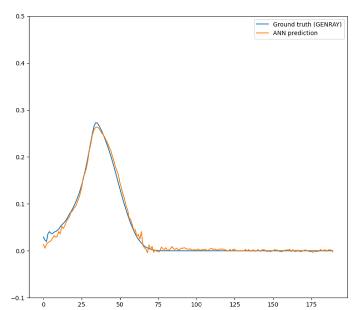
→ Larger Electron channel transport? Density Pump out?

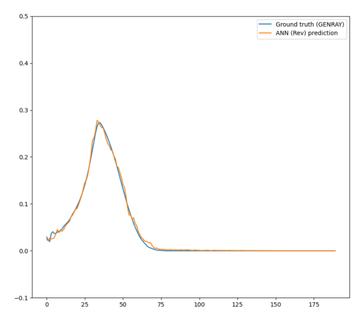






- Power absorption estimation by ML





[Jiho Park, IFPC conference (2024)]

Ongoing Project 2 – 4-D Fokker Planck code

- Development of 4-D FP heating and current drive code (FP4D) to parallel and drift physics (c.f. 3-D bounce-averaged FP, CQL3D)

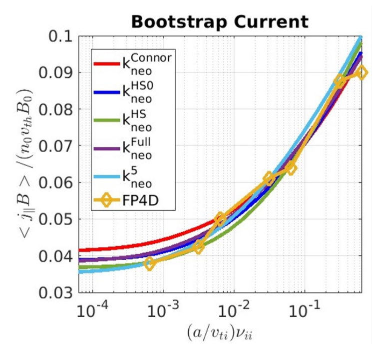
$$\frac{\partial f(r,\theta,v_\perp,v_\parallel)}{\partial t} + (\mathbf{v}_\parallel + \mathbf{v_D})\cdot\nabla f = C(f) + Q_{RF}(f) + E_{DC}(f) + S_{NB}(f) + \ldots$$

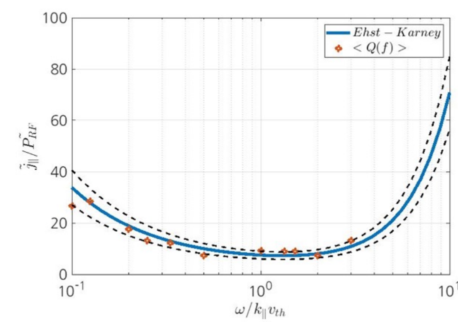
- Possible applications

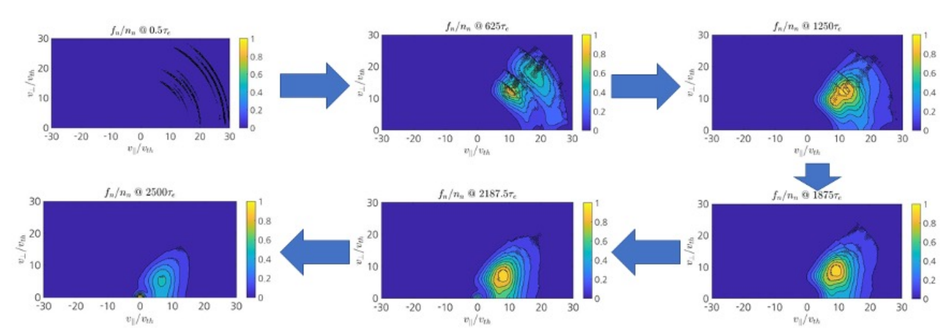
(1) Interplay between the neoclassical current and RF driven current

(2) Synergy between NB and RF

(3) Neoclassical transport change driven by RF (e.g. Tungsten impurity mitigation by ICRF)







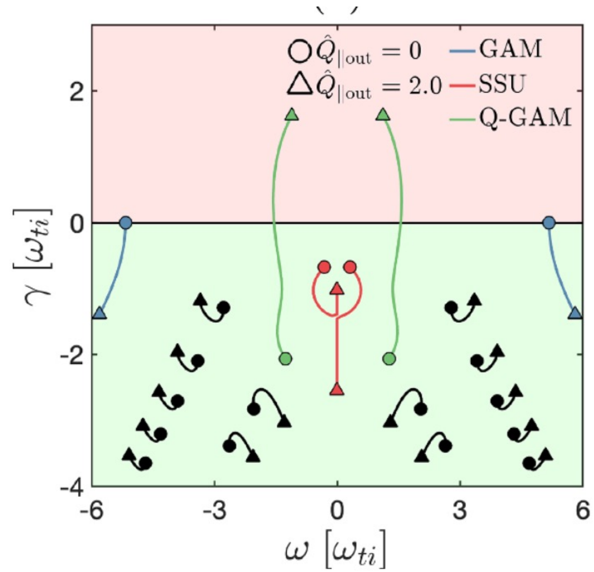
[Yunho Jeong, APS conference (2024)]

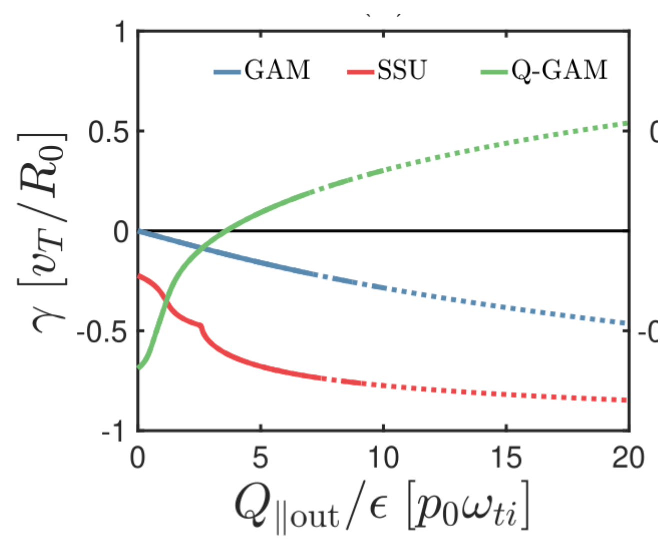
Ongoing Project 3 – Poloidal inhomogeneity in transport

- Effect of poloidal inhomogeneity in tokamak transport code – Development of ToTal-PI code

- A poloidal inhomogeneous particle source threshold for Hassam Spin-up instability [Young-Hoon Lee, et. al. PoP (2023)]

- A new GAM (QGAM) by poloidal inhomogeneous heat source [Young-Hoon Lee, et. al. NF (2024)]





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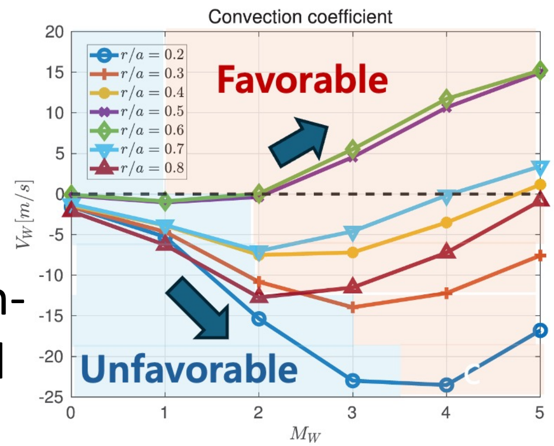
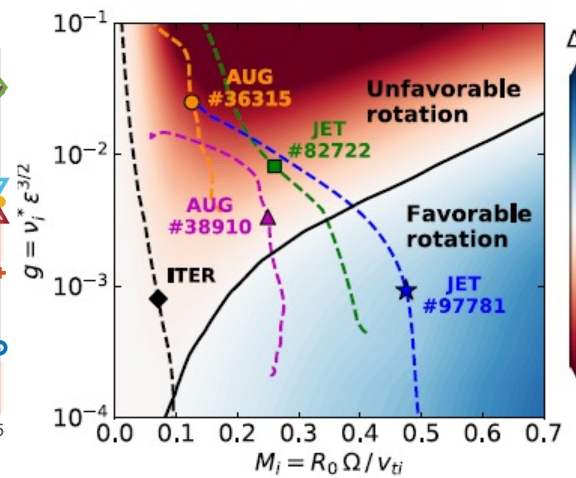
Ongoing Project 4 – Impurity transport

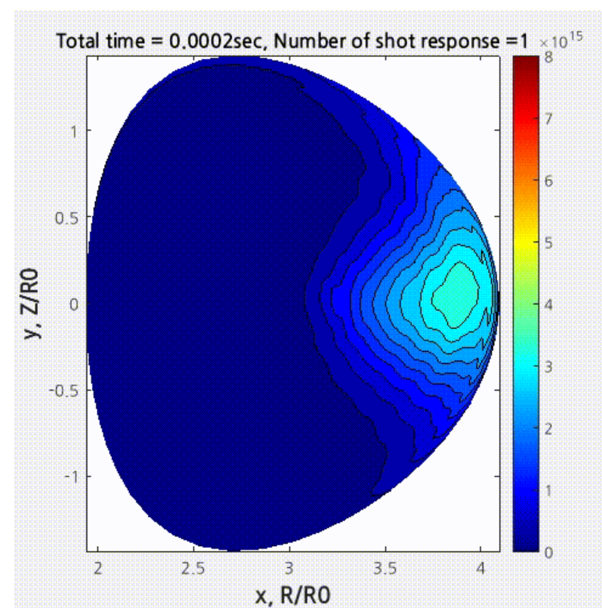
- KSTAR Tungsten injection experiment, and a new wall (2024)
- For a high flow (M_W~4), the non-monotonic dependency is found in the neoclassical transport
- 2-D impurity modeling with FACIT and TGLF in multi-time scale (short: parallel physics +long: transport) is being developed



[Hyojong Lee, PoP (2022)]



[Farjado, et. al. 2023 PPCF



Hyojong Lee, APS (2024)

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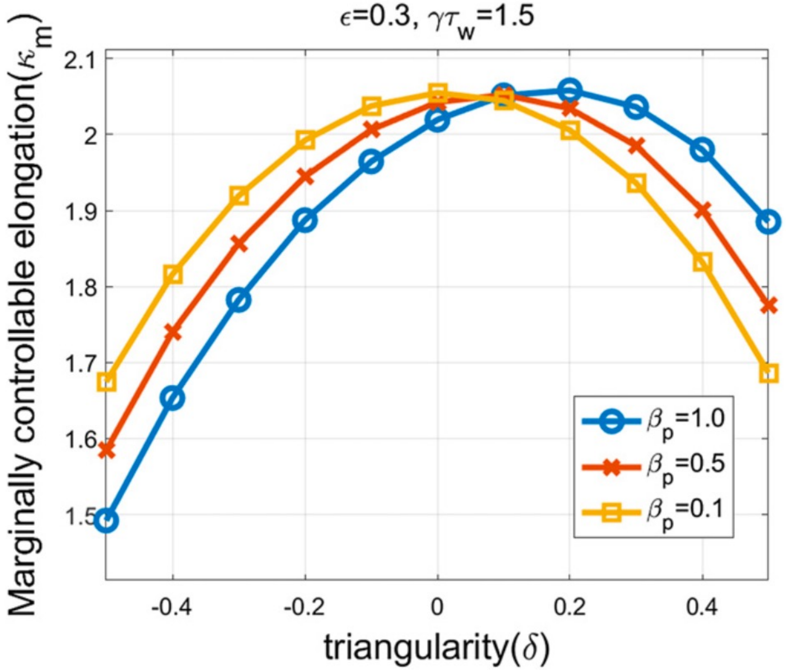
Ongoing Project 5 – MHD equilibrium/stability

- Tokamak MHD equilibrium code: ECOM [J.P. Lee, Cerfon, CPC (2015)]
- n=0 resistive wall mode for vertical instability: AVSTAB [J.P. Lee, Freidberg, NF (2017)]
- Analysis on the maximum elongation of negative triangularity plasmas (betap, li, wall)



[Junhyuck Song, Paz-soldan, J. P Lee, NF (2021)]

Entropy: A good fluid measure

- Boltzmann entropy is defined as $S(x,t) = -\int dv\, f\, \log f$

- Several good properties in a quasi-equilibrium system

(1)A state. Conservativity → $\mathrm{d}S(x,t)/dt = 0$, $\oint dS = 0,\ S(\mathrm{P},\mathrm{T})$

(2)Energy or heat exchange (1st thermodynamic law)

$$\delta Q = \langle \mathbf{E}\cdot\mathbf{J}\rangle = -T\int dv \log f_M q\mathbf{E}\cdot df/d\mathbf{v} = TdS/dt$$

(3)Irreversibility (2nd thermodynamic law)

-Force*Flux and quasilinear theory

-Boltzmann Collision (H-theorem)

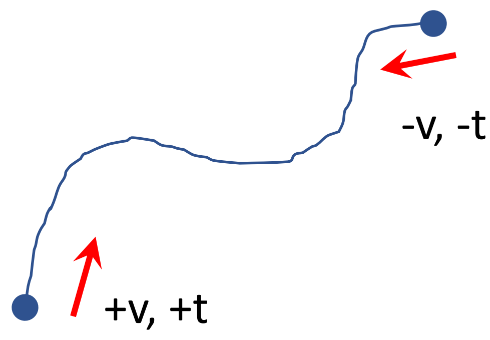
$$\frac{\partial S}{\partial t} \propto \Gamma\cdot F = \int dvD\left(\frac{\partial f}{\partial v}\right)^2 > 0$$

Time-reversal of Vlasov-Maxwell's equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_v f = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \left(q \int dv(vf) + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$



→ Time reversal with the opposite velocity also satisfies the equations

$$f' = f(-t, \mathbf{r}, -\mathbf{v}),\ \mathbf{E}' = \mathbf{E}(-t, \mathbf{r}) \text{ and } \mathbf{B}' = -\mathbf{B}(-t, \mathbf{r})$$

→ Entropy cannot change in the total system

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Entropy can change in a part of system

- Possible reasons to break the time-reversal in a given system (not total)

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_v (f_1 + f_0) = \nu f$$

(1) Collisions:
Krook collision operator → $f(-t, -v)$ changes only the left term, so cannot be a solution

(2) External source for distributions:
Energetic particles → f_0 is given

(3) External fields is given or dissipation is given → $\mathbf{B}' \neq -\mathbf{B}(-t, \mathbf{r})$

→ The wave-particle interactions can change the entropy of a small set of system (e.g. zero-k, or low-k), while its positiveness is not determined yet.

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Entropy Application on Fusion Plasma Physics

- MHD equilibrium by maximizing magnetic field energy [Taylor (1974)]
- Free energy balance for drift/MHD turbulence [Sugama (1996), Schekochihin (2009)], Self-organized transport, Turbulent equipartition [Yankov (1994), Hahm (2007)]
- A large system efficiency [Hasegawa (2014)] – EC input, Alfven wave output?

"A new concept of a fusion device as an open system is presented in which the desirable plasma structure is produced by continuous injection of negentropy (or free energy) from and ejection of increased entropy to external environment."

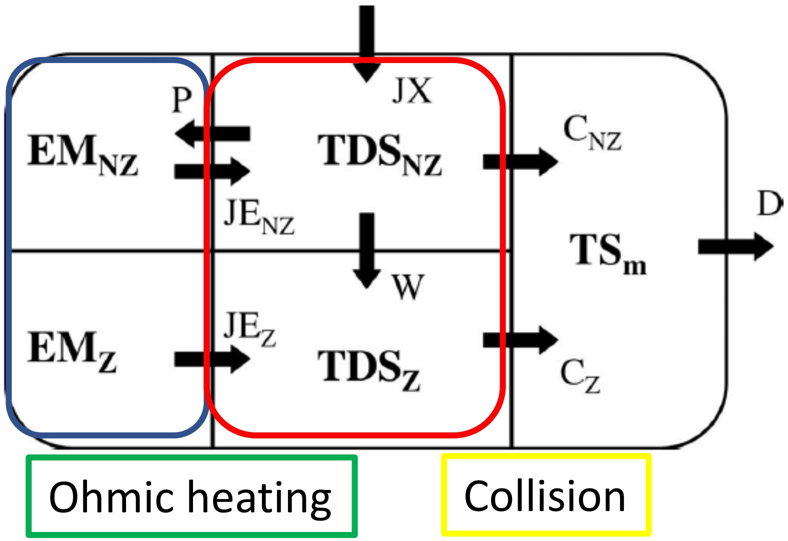
- Controversy of usefulness for an open, nonlinear, non-equilibrium system.

(1) Transport by entropy balance [Property 2] → Chapter 2

(2) Nonlinear saturation of Energetic particle instability (Berk-Breizman model) by maximizing entropy [Property 2 and 3] → Chapter 3

Free energy balance for drift-kinetic turbulence

- Turbulence by zonal and non-zonal components [Sugama (1996, 2009)]



$$T\frac{dS_{k1}(x,t)}{dt} = \int dv f_{k_1}\left(E_{k_1-k_2}\frac{\partial f_{k_2}}{\partial v}\right)$$

$$= -T\frac{dS_{k2}(x,t)}{dt} = -\int dv f_{k_2}\left(E_{k_2-k_1}\frac{\partial f_{k_1}}{\partial v}\right)$$

Region	Quantity
TS$_m$	$-\Sigma_a T_a\langle\langle\int d^3v(f_{aM}+\delta f_a)\log(f_{aM}+\delta f_a)\rangle\rangle$
TDS$_{NZ}$	$\Sigma_{\mathbf{k}_\perp(NZ)}\Sigma_a T_a\langle\langle\int d^3v\lvert\delta f_{a\mathbf{k}_\perp}\rvert^2/2f_{aM}\rangle\rangle$
TDS$_Z$	$\Sigma_{\mathbf{k}_\perp(Z)}\Sigma_a T_a\langle\langle\int d^3v\lvert\delta f_{a\mathbf{k}_\perp}\rvert^2/2f_{aM}\rangle\rangle$
EM$_{NZ}$	$\Sigma_{\mathbf{k}_\perp(NZ)}\langle\langle\lvert\mathbf{E}_{\mathbf{k}_\perp}\rvert^2+\lvert\mathbf{B}_{\mathbf{k}_\perp}\rvert^2\rangle\rangle/8\pi$
EM$_Z$	$\Sigma_{\mathbf{k}_\perp(Z)}\langle\langle\lvert\mathbf{E}_{\mathbf{k}_\perp}\rvert^2+\lvert\mathbf{B}_{\mathbf{k}_\perp}\rvert^2\rangle\rangle/8\pi$

Arrow	Quantity
JX	$\Sigma_a T_a(J^A_{a1}X^A_{a1}+J^A_{a2}X^A_{a2})$
W	$\mathcal{T}$(NZ→Z) [Eq. (69)]
C$_{NZ}$	$-\Sigma_{\mathbf{k}_\perp(NZ)}\Sigma_{a,b}T_a\langle\langle\int d^3v(\delta f^*_{a\mathbf{k}_\perp}/f_{aM})C^L_{ab}(\delta f_{a\mathbf{k}_\perp},\delta f_{b\mathbf{k}_\perp})\rangle\rangle$
C$_Z$	$-\Sigma_{\mathbf{k}_\perp(Z)}\Sigma_{a,b}T_a\langle\langle\int d^3v(\delta f^*_{a\mathbf{k}_\perp}/f_{aM})C^L_{ab}(\delta f_{a\mathbf{k}_\perp},\delta f_{b\mathbf{k}_\perp})\rangle\rangle$
D	$\Sigma_a(T_a/V')(\partial/\partial s)[V'\{(S_{aM}/n_a)J^A_{a1}+J^A_{a2}\}]$
P	$-(c/4\pi V')(\partial/\partial s)[V'\langle\langle(\mathbf{E}\times\mathbf{B})\cdot\nabla s\rangle\rangle]$
JE$_{NZ}$	$\Sigma_a e_a n_a\Sigma_{\mathbf{k}_\perp(NZ)}\mathrm{Re}\langle\langle(\mathbf{u}^*_{a\mathbf{k}_\perp}\cdot\mathbf{E}_{\mathbf{k}_\perp})^{(3)}\rangle\rangle$
JE$_Z$	$\Sigma_a e_a n_a\Sigma_{\mathbf{k}_\perp(Z)}\mathrm{Re}\langle\langle(\mathbf{u}^*_{a\mathbf{k}_\perp}\cdot\mathbf{E}_{\mathbf{k}_\perp})^{(3)}\rangle\rangle$

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Free energy reservoir is captured

- For the plasma perturbation by electromagnetic fields, the field and kinetic energy can be expressed by [Kruskal and Oberman (1958), Morrison (1990)]

$$H = -\frac{m}{2}\int \frac{v(\delta f)^2}{df/dv}dvdx + \frac{\epsilon_0}{2}\int \delta\phi_x^2 dx$$
$$= V\frac{\epsilon_0}{4}\frac{\partial(\omega\epsilon_r)}{\partial\omega}\|\mathbf{E}(k,\omega)\|^2$$

- Non-resonant wave-plasma interaction can be captured in the entropy part

- Related with the non-linear saturation by the sub-dominant modes [Hatch (2011)] instead of the saturation by zonal flow shearing

- Nonlinear saturation by negative mode [Fried (1971), Dewar (1973)] $\frac{\partial\epsilon}{\partial\omega} < 0$

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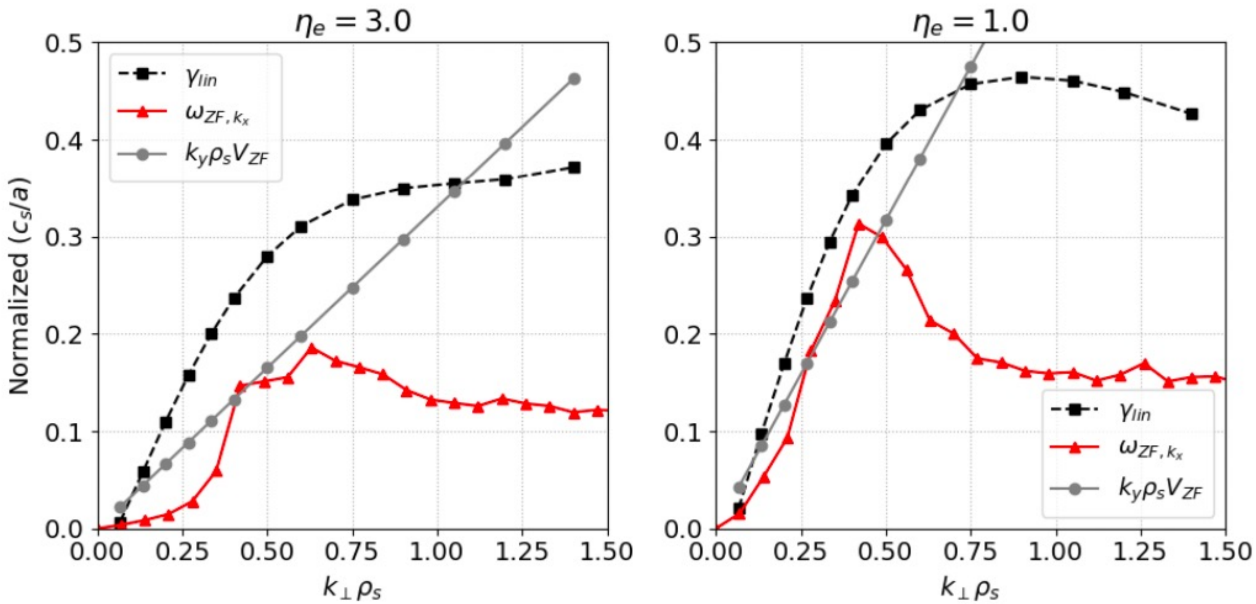
Some TEM turbulence show weak zonal flow interplay

- **Trapped electron mode (TEM) cases in CGYRO nonlinear simulations**



- **$E \times B$ shearing rate(ω_{ZF,k_x})[1] shows that zonal flow saturation is weak in $\eta_e = 3$. ($\gamma_{lin} \sim 0.5\omega_{ZF,k_x}$)**

- **TGLF saturation model assumed $V_{ZF} = \gamma_{lin,max}/k_y$ [2] which is not satisfied in $\eta_e = 3$.**

Zonal flow velocity : $V_{ZF} = \sqrt{\sum_{k_x} dk_x |k_x \Phi_{k_x}^2|}$

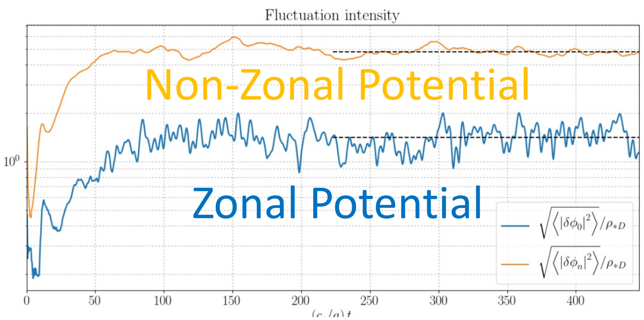
1] R. E. Waltz and C. Holland (2008) Phys. Plasmas 15, 122503
2] G. M. Staebler et al. (2016) Phys. Plasmas 23, 062518

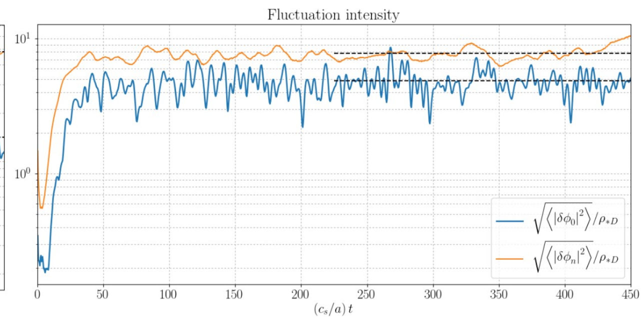
Some TEM turbulence show weak zonal flow interplay

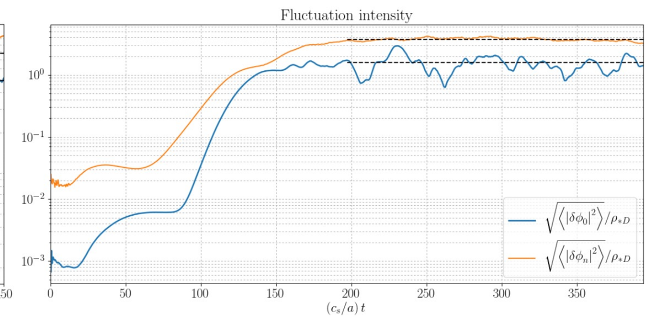
- **<u>Weak ZF</u> in Trapped electron mode (TEM) case:** $T_e/T_D = 2\ ,\ \eta_e = 3(a/L_n = 1\ ,\ a/L_{T_e} = 3)$

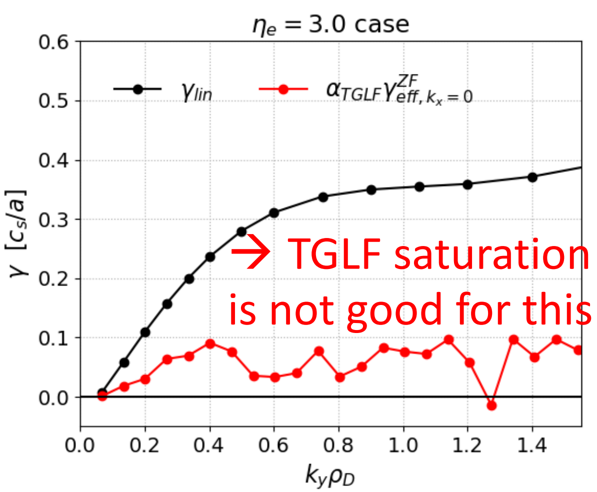
  → Electron heating dominated case

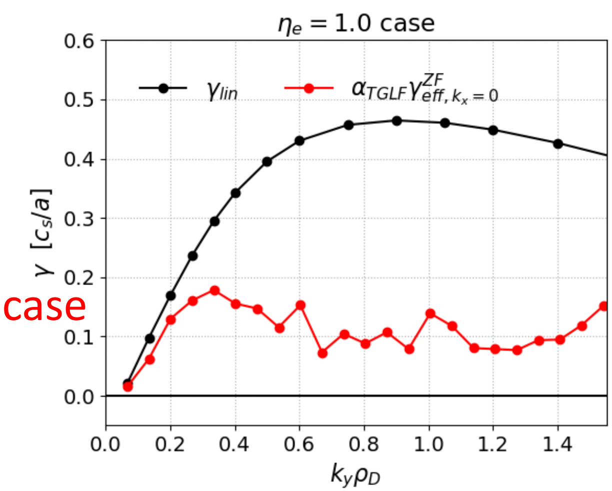
TEM ($\eta_e = 3$) | **TEM ($\eta_e = 1$)** | **ITG (STD)**

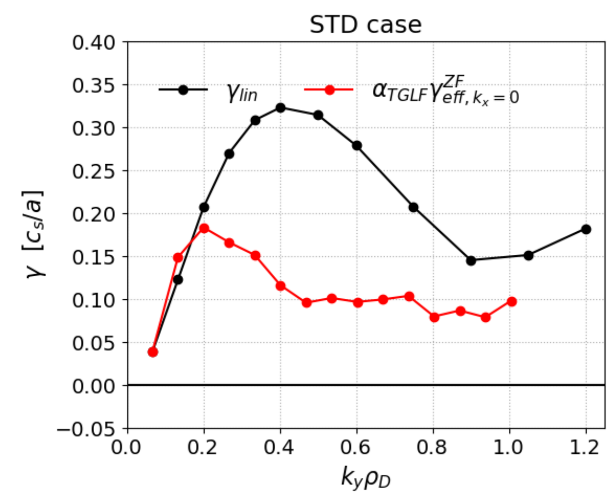












Where the effective growth rate by ZF $\gamma^{ZF}_{eff,k_\perp}$

$$\gamma^{ZF}_{eff,k_\perp} = \sum_a t_{a,k_\perp} / 2 \sum_a \delta S_{a,k_\perp}$$

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Excitation: Strong electron channel $T_{ZF,e}$

- Velocity space integral and field line average of G.K. eq. [Sugama (2009), Nakata (2012)]

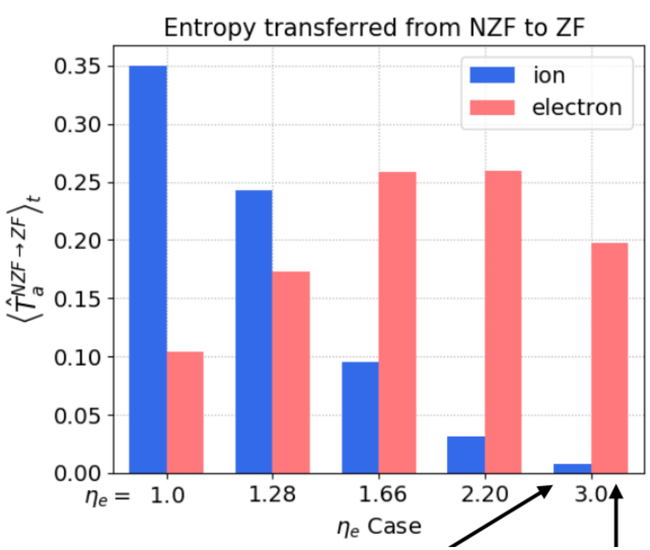
<Free energy balance>

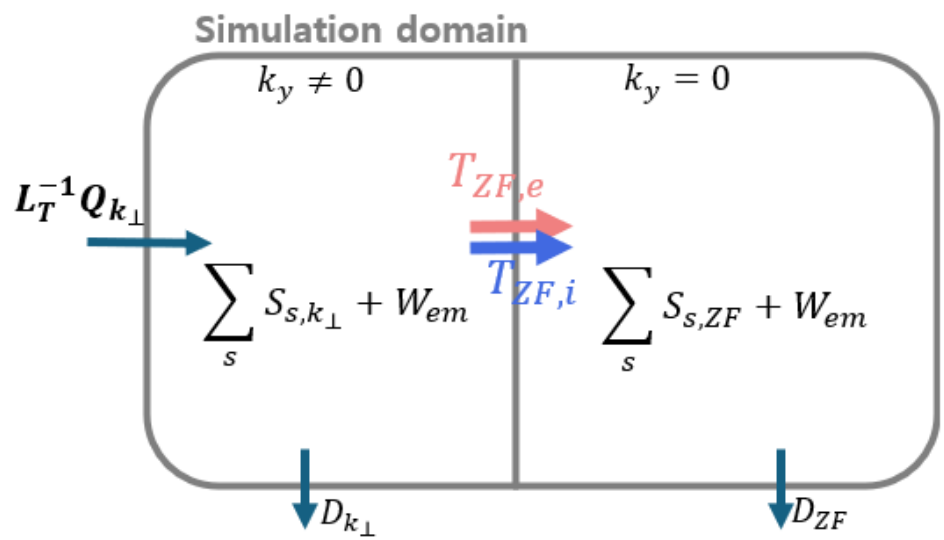
$$\frac{d}{dt}\left(\sum_s S_s + W_{em}\right)_{ZF} = \sum_s T_{ZF,s} + D_{ZF}$$

- Nonlinear entropy transfer from non-zonal flow components to zonal flow (ZF)

$$T_{ZF,s} = \sum_{r\epsilon ZF}\sum_{p\epsilon NZF}\sum_{q\epsilon NZF} T_s \left\langle \frac{c}{B} b\cdot k_p \times k_q \int \frac{1}{2F_{0s}} Re[\delta\psi_p h_{s,q} h_{s,r} - \delta\psi_q h_{s,p} h_{s,r}]\, d^3v \right\rangle$$

- Where ZF excitation is weak, $T_{ZF,e}$ largely remains.





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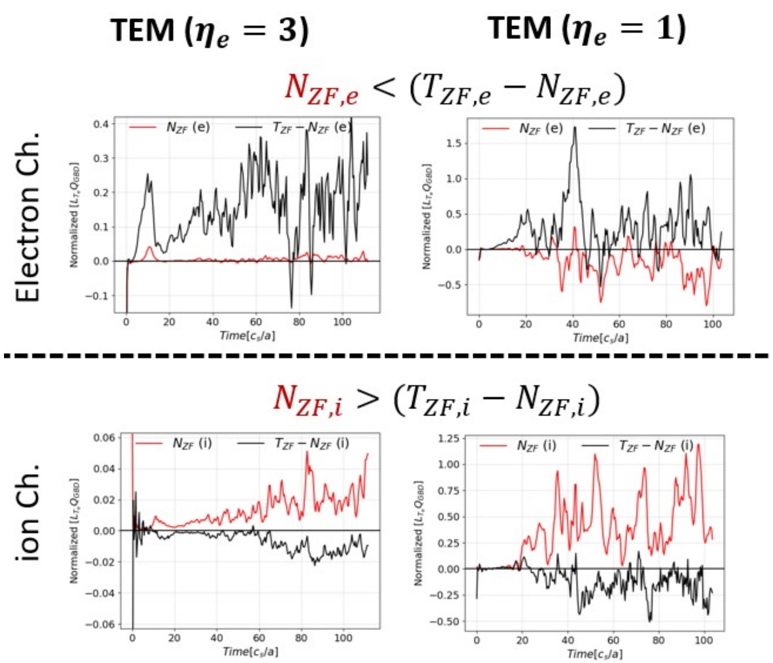
Excitation: T_{ZF} does not excite the fields much

- Entropy balance eq. can be compared with the Poisson eq. [Ishizawa (2019)]
- For high η_e, the contribution to the field N_{ZF} by electron channel is small. Most entropy is transferred to the kinetic part.

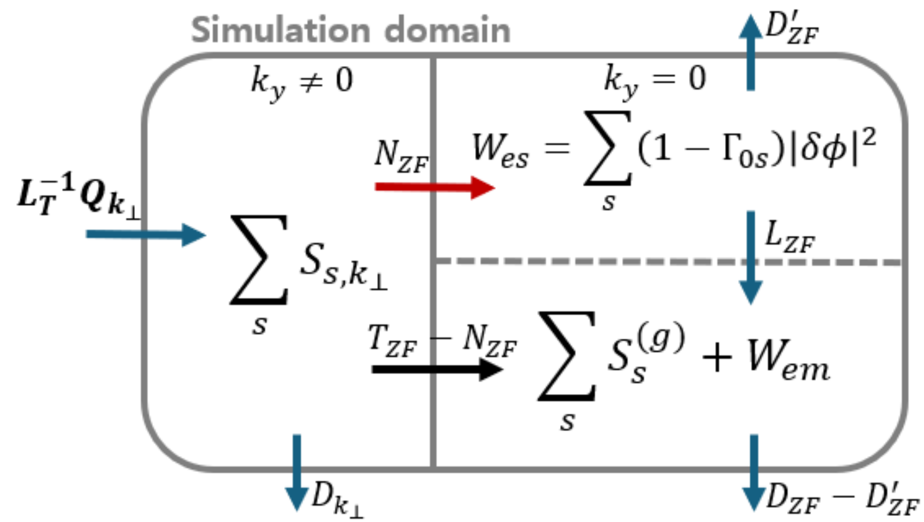


<Vorticity modeling> [Chen (2022)]

$$\chi \frac{d}{dt}\phi_{ZF} = g(a_+\phi_0\phi_+ - a_-\phi_0\phi_-)$$

<Field energy balance>

$$\frac{\epsilon_0}{2}\frac{d}{dt}\phi_{ZF}^2 = \frac{d}{dt}W_{es} = \sum_s N_{ZF} + \sum_s L_{ZF} + D'_{ZF}$$



[Jiheon Song, PRE (to be submitted)]

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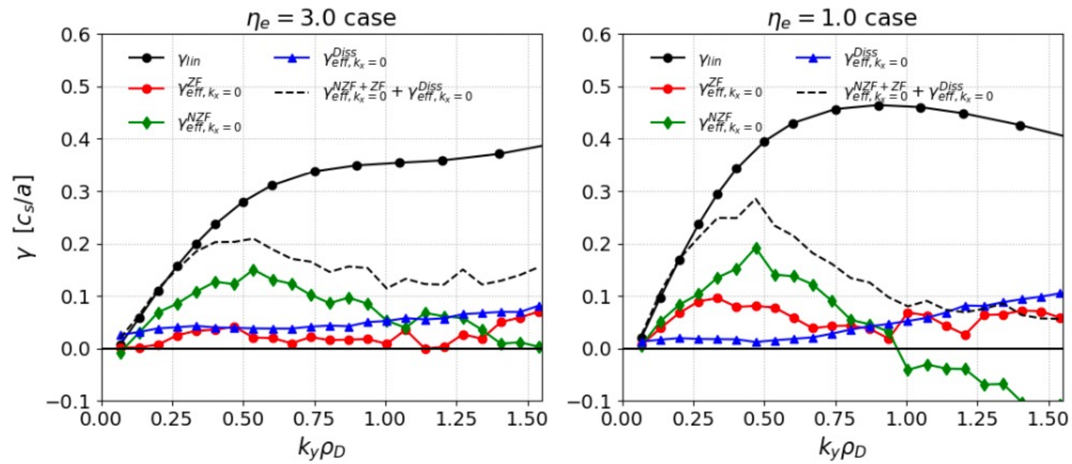
Saturation: Energy transfer to Non-zonal components

- In terms of the free energy,

$$\gamma_{eff} = \gamma_{eff}^{ZF} + \gamma_{eff}^{NZF} + \gamma_{eff}^{Diss}$$

- High ratio of γ_{eff}^{NZF} stands for the perpendicular diffusion for high $\boldsymbol{\eta_e}$
- Quantitative ratio can be useful for the saturation model

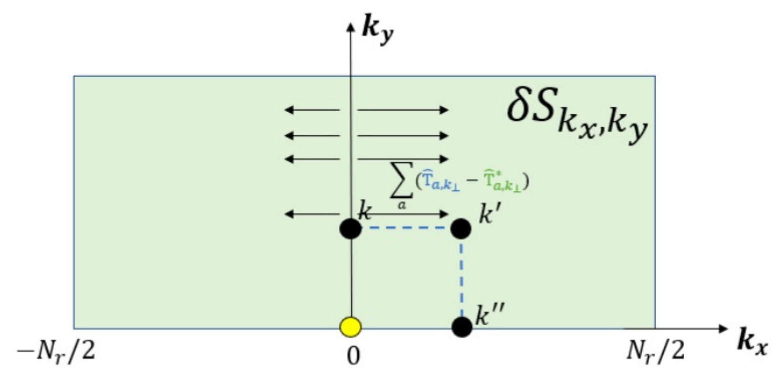


$$\gamma_{eff} = \frac{1}{2\delta S_{k_\perp}} \frac{\delta S_{k_\perp}}{dt}\Big|_{NL} = \frac{\sum_a \hat{T}_{a,k_\perp}}{\sum_a 2\delta \hat{S}_{a,k_\perp}} \quad \text{(eff. growth rate)}$$

$$\gamma_{eff}^{ZF} = \frac{\sum_a (\hat{T}_{a,k_\perp} - \hat{T}^*_{a,k_\perp})}{\sum_a 2\delta \hat{S}_{a,k_\perp}} \quad \text{(eff. growth rate by ZF)}$$

$$\gamma_{eff}^{NZF} = \frac{\sum_a \hat{T}^*_{a,k_\perp}}{\sum_a 2\delta \hat{S}_{a,k_\perp}} \quad \text{(eff. growth rate by NZF)}$$

$$\gamma_{eff}^{Diss} = \frac{\sum_a \hat{D}_{a,k_\perp}}{\sum_a 2\delta \hat{S}_{a,k_\perp}} \quad \text{(eff. growth rate by Dissipation)}$$



$$\hat{T}_{a,k_\perp} = \sum_{k'} \sum_{k''} T_a \left\langle \int d^3v \frac{c}{B} \boldsymbol{b} \cdot (k'_\perp \times k''_\perp) Re\left[\psi_{a,k'_\perp} H_{a,k''_\perp} H_{a,k_\perp}\right] \right\rangle / Q_{GBD} L_{T_e}^{-1} \quad \text{(Entropy transfer rate with ZF mediated)}$$

$$\hat{T}^*_{a,k_\perp} = \sum_{k' \in NZF} \sum_{k'' \in NZF} T_a \left\langle \int d^3v \frac{c}{B} \boldsymbol{b} \cdot (k'_\perp \times k''_\perp) Re\left[\psi_{a,k'_\perp} H_{a,k''_\perp} H_{a,k_\perp}\right] \right\rangle / Q_{GBD} L_{T_e}^{-1} \quad \text{(Entropy transfer rate w/o ZF mediated)}$$

Positiveness of turbulent diffusion

- As in neoclassical transport, the turbulent transport by linear gyrokinetic equations has the Onsager symmetry [Sugama and Horton (1998)],

$iG_{is}\widetilde{H}_{is} = i\sum_{ix} W_{is,ix}X_{is,ix}$ where $X_{is,ix} = \left\{-\frac{\partial \ln n_{is}}{\partial r}, -\frac{\partial \ln T_{is}}{\partial r}, \dots\right\}$

$$F_{is} = Re\left[\langle -in_{is}\widetilde{H}_{is}^{*}W_{is,ix}\rangle\right] = \sum_{jx} L_{is,ix,jx} X_{is,jx}$$

$$L_{is,ix,jx} = Re\left[\left\langle -i\left(W_{is,jx}G_{js}^{-1}\right)^{*} W_{is,ix}\right\rangle\right]$$

→ Hermition matrix does not gurantee the positiveness of the single diffusion coeff.

$$L_{is,ix,jx}\left(\boldsymbol{B}, \boldsymbol{V}_{\mathbf{0}}, \phi(t)\right) = L_{is,jx,ix}\left(-\boldsymbol{B}, -\boldsymbol{V}_{\mathbf{0}}, \phi(-t)\right)$$

→ Time dependency can be ignorable for a bounce average

- With the Hammett-Perkins closure in Gyro-Landau-fluid code, the symmetry is broken
→ Corrected form the positive-definiteness in the coefficients of the moments [Staebler et. al (2023)]

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Quasilinear Onsager symmetry

- If a resonance broadening kernel is considered, the diffusion coefficient could be positive. in QuaLiKiz [Bourdelle (2016)]

$$\Gamma_s = \sum_{\boldsymbol{n},\omega} Im\left[\frac{1}{\omega - \boldsymbol{n}\cdot\Omega_J + i\nu_{QL}}\right]\boldsymbol{n}\cdot\frac{df_{0,s}}{dJ}e_s^2|\phi|^2$$

$$\nu_{QL} \simeq \gamma$$

- For Kubo number<1, the decorrelation time is shorter than the eddy turn over time → Valid random walk model . The QL approach shows 15% RMS error in temperature profile reconstruction [Bourdelle (2016)]

- Then, how much does the nonlinear coupling affect the positiveness? Any kinetic structure, which is preferential to the positiveness?

Nonlinear saturation of energetic particle instability

- Berk-Briezman model: A 1-D model explaining the nonlinear saturation for an instability by the energetic particles by dissipation

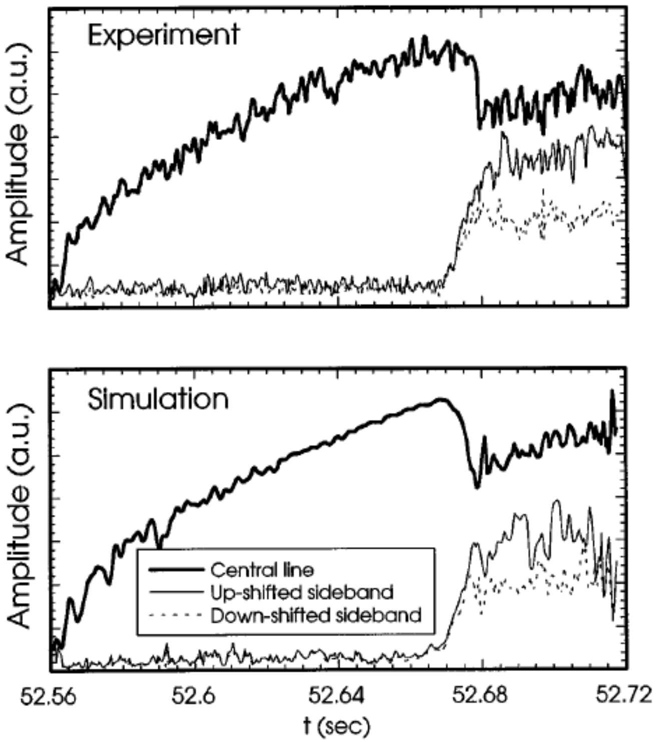
[Berk, Briezman, Pekker (1996)]

- Diffusion and drag in collision

[Lilley et. al. (2009), Duarte (2017)]

- Frequency Chirping.

[Berk (1999), Duarte (2019)]



<Comparison between JET TAE measurement and BB model [Fasoli (1998)]

$$f_1 = \int_{t'}^{t} ds_1\, e^{iu(t-s_1)+\nu(t-s_1)} \frac{1}{4}\omega_B^{2*}(s_1) \frac{\partial(F_0 + f_0(s_1))}{\partial u}$$

and

$$\frac{\partial \omega_B^2}{\partial t} = -\frac{e\omega}{\epsilon_0 mk} \int f_1 dv - \gamma_D \omega_B^2$$

where $\omega_B = (ekE/m)^{1/2}$

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Entropy partition and energy balance

- Entropy can be partitioned for each Fourier component

$$\frac{dS_s}{dt} = -\int dv \frac{\partial F_s \log F_s}{\partial t} = -\int dv \frac{\partial}{\partial t}\left((F_0+f)(\log F_0 + \frac{f}{F_0} - \frac{f^2}{2F_0^2} + O(\epsilon^3))\right)$$

$$= -\int dv (1 + \log F_0 + \frac{f}{F_0})\frac{\partial f}{\partial t} \equiv \frac{d}{dt}(S_0^{(2)} + S_0^{(4)} + S_1^{(2)} + S_{-1}^{(2)} \dots),$$

- Entropy change by wave-particle interaction and collisions can be partitioned

$$\sigma_c^{(a,b)} = -\int dv f_c^{(a)} \dot{f}_c^{(b)}$$

(a+b) is the order of the electric fields (not necessary)

$$\frac{\partial S_0^{(2)}}{\partial t} = -\int dv \log F_0 \frac{\partial f_0}{\partial t} \simeq \left(\sigma_0^{Q(0,2)} + \sigma_0^{Q(0,4)} + \sigma_0^{C(0,2)}\right)$$

$$\frac{\partial S_0^{(4)}}{\partial t} = -\int dv \frac{1}{2F_0}\frac{\partial f_0^2}{\partial t} \simeq \left(\sigma_0^{Q(2,2)} + \sigma_0^{C(2,2)}\right)$$

$$\frac{\partial S_1^{(2)}}{\partial t} = -\int dv \frac{1}{2F_0}\frac{\partial f_1^2}{\partial t} \simeq \left(\sigma_1^{Q(1,1)} + \sigma_1^{Q(3,1)} + \sigma_1^{Q(1,3)} + \sigma_1^{C(1,1)}\right),$$

Field evolution equation (Ampere's law)

$$\frac{\partial \hat{U}_E}{\partial \tau} + \hat{\sigma}^D = \left(\hat{\sigma}_0^{Q(0,2)} + \hat{\sigma}_0^{Q(0,4)}\right)$$

where $\hat{U}_E = \frac{\widehat{\omega}_b^4}{4\pi}$ and

$$\hat{\sigma}_D = \gamma_d \frac{\widehat{\omega}_b^4}{2\pi}$$

Free energy balance given by F0 and fields

- A new equation for the free energy balance is defined by

$$\frac{\partial}{\partial\tau}(\hat{U}_E - \hat{S}_0^{(2)}) = \hat{\sigma}^S - \hat{\sigma}^D$$

where $\hat{\sigma}^S = -\hat{\sigma}_0^{C(0,2)} = \left(2\int dv f_0^{(2)}(S_p)/F_0\right)\left(\frac{\gamma_L}{c_0}\frac{\gamma_L}{k}\left(\frac{m\gamma_L^2 k}{2\pi|e|^2\omega_{pe}}\right)^2\right)^{-1}$ is the entropy source from energetic particle

- Normalized entropy $-\hat{S}_0^{(2)}$ represent kinetic energy, while the change of total entropy of the non-oscillating mode is given by $S_0^{(2)} + S_0^{(4)}$

$$\frac{dH}{dt} = \int dv \frac{m}{2}\left(\frac{\omega_{pe}}{k} + v\right)^2 \frac{\partial f_0}{\partial t} \simeq m\frac{\omega_{pe}}{k}\int dv v \frac{\partial f_0}{\partial t} = T_{BB}\frac{\partial S_0^{(2)}}{\partial t}$$

- $T_{BB} = -2m(c_0/c_1)(\omega_{pe}/k)$ is an equivalent temperature for BB model due to the given distribution $F_0 = c_0 + c_1(v - \frac{\omega_{pe}}{k})$

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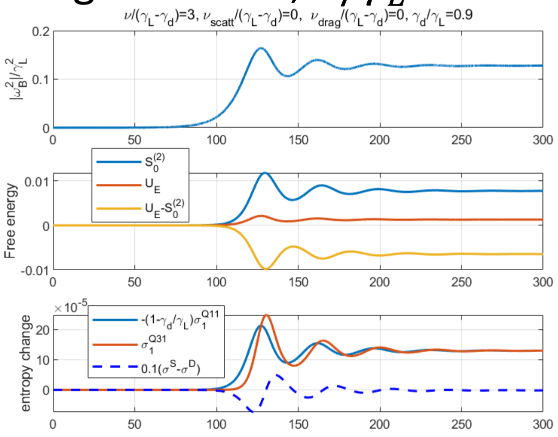
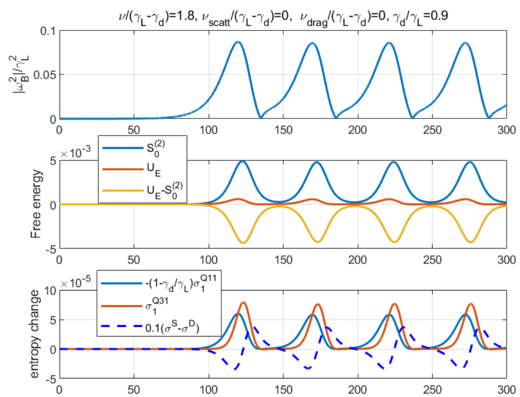
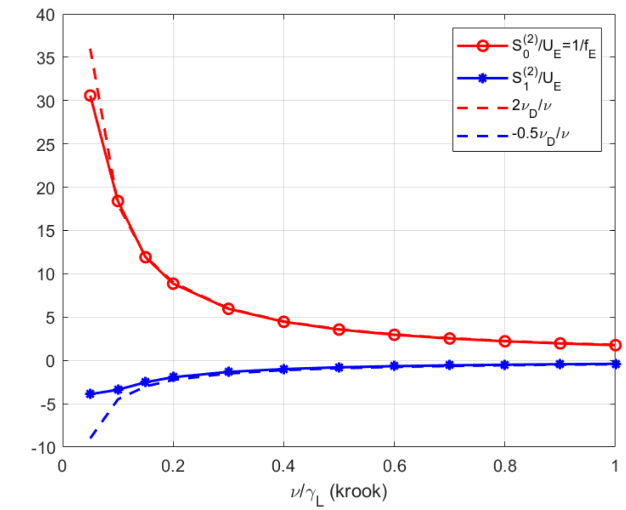
Kinetic energy decreases and electric fields increases

<high collision, $\nu/\gamma_L = 0.3$>



<low collision, $\nu/\gamma_L = 0.18$>





- In a steady state, $\hat{S}_0^{(2)}$ increase (kinetic energy decrease), giving the field energy U_E increase
- Saturation occurs when ($\hat{\sigma}_0^{Q(02)} + \hat{\sigma}_0^{Q(04)} = \hat{\sigma}_1^{D}$)
- In a periodic case, $\hat{S}_0^{(2)}$ and U_E are also periodic.

- The low ratio of $\hat{S}_0^{(2)}/U_E$ for the high collision is good for high conversion from kinetic energy to fields (e.g. alpha channeling)

Entropy equipartition

- For two microstates for n particles, maximizing possible states numbers (entropy)

$$S \propto \log\left(\frac{\mathrm{n!}}{(\mathrm{xn})!((1-\mathrm{x})\mathrm{n})!}\right) = \text{-}x\text{log}x\text{+(1-x)log(1-x)}$$

results in *x=0.5* when the number of particle is constrained.

- Maximizing entropy $\mathrm{L} = -\int dv\,(f\log f) + \lambda(\int dv\, f - n)$

- with the Lagrange multiplier for density constraints → uniform distribution
- with the Lagrange multiplier for energy and density constraints → Maxwellian distribution

- The reason of the maximum entropy can be originated from H-theorem in the quasi-equilibrium $\frac{\mathrm{d}S}{dt} > 0$

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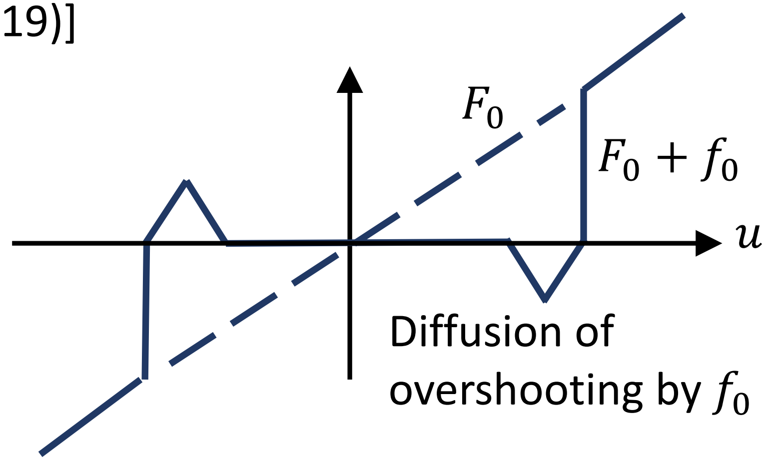
Quasilinear (bilinear) diffusion in a velocity space

- For the wave-particle interaction, the bounce-averaged quasilinear diffusion is positive-definite [Kaufman (1972), Jungpyo Lee (2019)]

$$\frac{\partial f_0}{\partial t} = \left(\frac{-\omega_B^2}{2}\frac{\partial f_1^*}{\partial u} + \frac{-\omega_B^{2*}}{2}\frac{\partial f_1}{\partial u}\right) + C(f_0)$$

$$\frac{\partial f_1}{\partial t} + iuf_1 = \frac{-\omega_B^2}{2}\frac{\partial (F_0+f_0)}{\partial u} - \frac{\omega_B^{2*}}{2}\frac{\partial f_2}{\partial u} + C(f_1)$$



$$\rightarrow \frac{\partial f_0}{\partial t} = \frac{1}{4}\omega_B^{2*}(t)\frac{\partial}{\partial u}\left(\int_{t'}^{t} ds\, e^{iu(t-s)+\nu(t-s)}\, \omega_B^2(s)\frac{\partial(F_0+f_0+f_2)}{\partial u} + f_1(t')e^{iu(t-t')+\nu(t-t')}\right) + C(f_0)$$

$$\rightarrow \left\langle \frac{d(S_0^{(2)}+S_0^{(4)})}{dt}\right\rangle_b \simeq \left(\int_{t-t_{dec}}^{t} dt\, e^{-iut}\, \omega_B^2(t)\frac{\partial(F_0+f_0)}{\partial u}\right)^* \left(\int_{t-t_{dec}}^{t} ds\, e^{-ius}\, \omega_B^2(s)\frac{\partial(F_0+f_0)}{\partial u}\right) \geq 0$$

- Invalidating the positiveness could be due to the collision in phase, weak decorrelation from $f_1(t')$, and the contribution from f_2

Frequency Chirping = Flattening + Collapse + Energy Dissipation

- Process of the frequency chirping [Berk (1997) PLA]

→ Like BGK mode, action variable can be used by $g(J) = f(J) - F_0(\omega_0 + \Omega_1)$

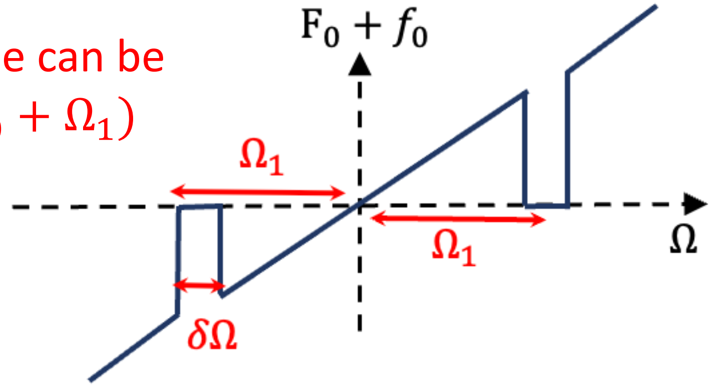
(1) Plateau is formed

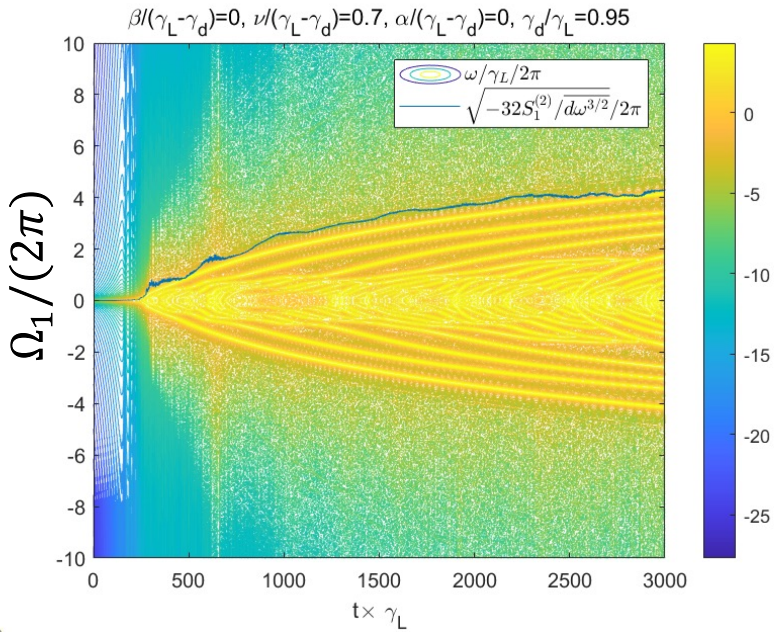
(2) As the plateau width becomes large, the sharp gradient makes the negative wave mode makes the collapse. → $\frac{\partial \epsilon}{\partial \omega}<0$

(3) The energy dissipation make the move of the plateau

→ $\frac{\partial g(J)}{\partial t} = C(g) - \frac{d\Omega_1(J)}{dt}\frac{\partial F_0(\omega_0)}{\partial \Omega}$

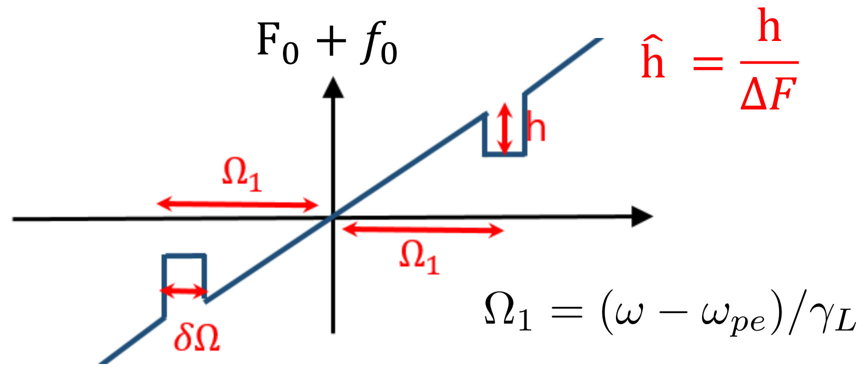
→ $\frac{d\Omega_1(J)}{dt} \propto \sqrt{t}$





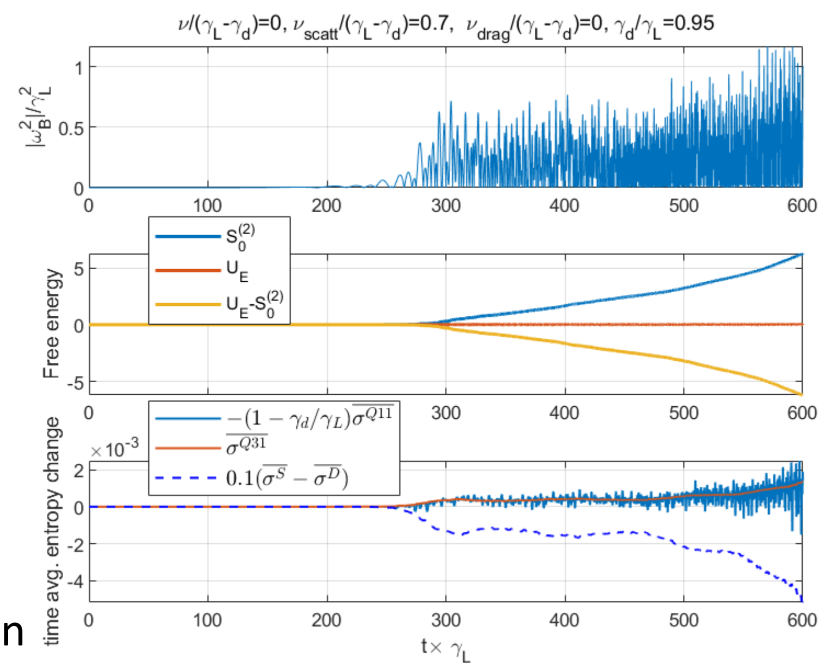
Semi-adiabatic chirping with $\hat{h} \neq 1$

- For a low collision $\nu/(\gamma_L - \gamma_d) < 0.8$, the hole and clump formation and frequency shift
- Adiabatic action-angle variable analysis results in $\hat{h} = 1$ and $\Omega_1^2 \propto \gamma_d t$ [Berk (1999)]



- For the chirping case, the entropy balance still works by time average

$$\frac{\partial}{\partial \tau}(\hat{U}_E - \hat{S}_0^{(2)}) = \hat{\sigma}^S - \hat{\sigma}^D$$



BOT [Lilley (2009)] simulation

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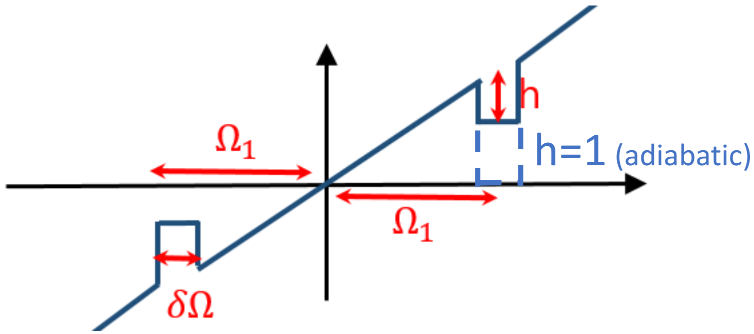
Finding $\hat{h}$ by the entropy maximization

< Variational method for Equipartition>

- Objectives: Using the model, find $\hat{h}$ and Ω_1, $\delta\Omega$ by maximizing the entropy of $S_0^{(2)} + S_0^{(4)}$



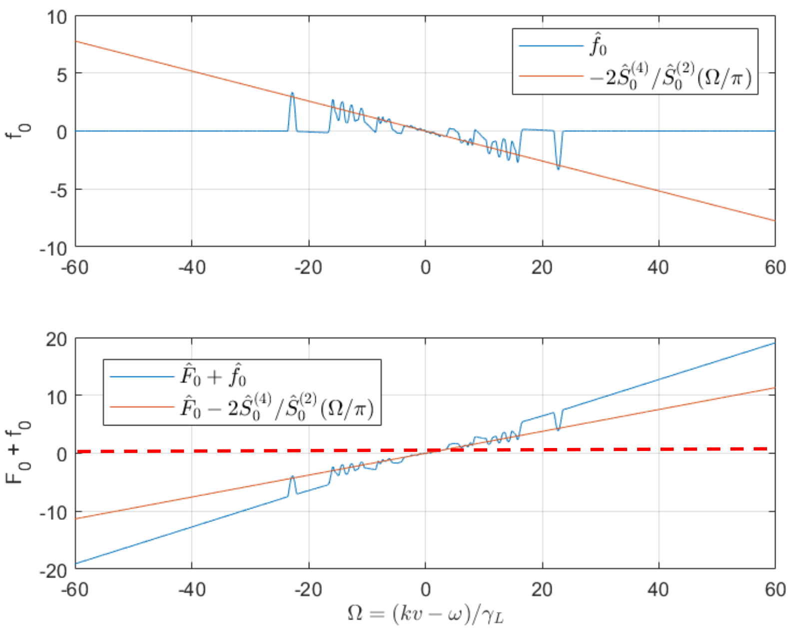
Three constraints by Lagrange multipliers:
(1) Particle number
(2) Energy balance (Long time response by dissipation)
(3) Frequency change (Short time response by Ampere's law)

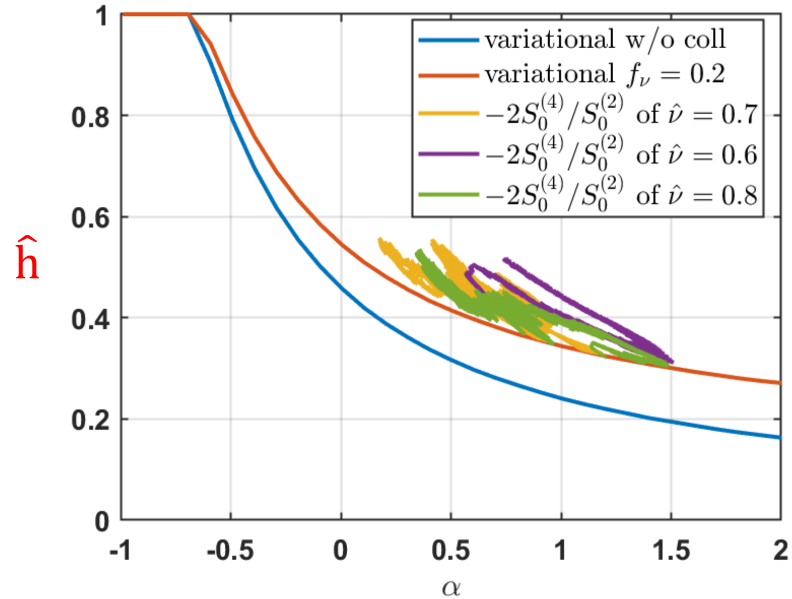
$$\hat{L} = \hat{S}_0^{(2)} + \hat{S}_0^{(4)} + \lambda_1\left(\int dv f_0\right) + \lambda_2\left(\hat{S}_0^{(2)} - \int dt\sigma_D\right) + \lambda_3(\omega^2 E^2 - (4\pi)^2 J^2)$$

$$\hat{S}_0^{(2)} \simeq 2\hat{h}(\Omega_1/\pi)^2\delta\Omega$$
$$\hat{S}_0^{(4)} \simeq -\hat{h}^2(\Omega_1/\pi)^2\delta\Omega$$

Agreement between simulations and variational method

- Using BOT simulations, the results for the average ĥ agrees with the variational method results





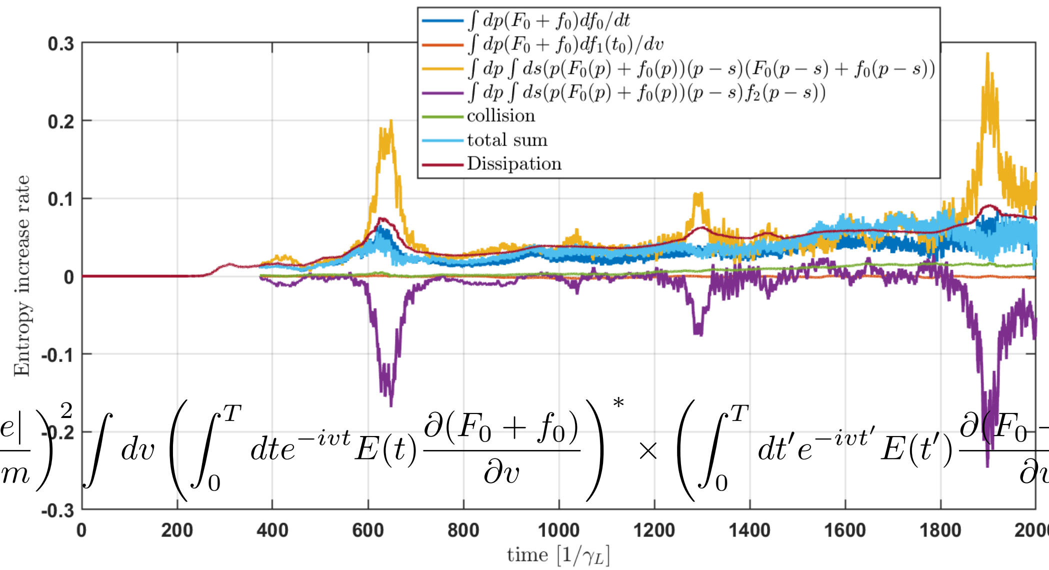
α is the ratio between the entropy transfer from n=1 to n=2 to one from n=1 to n=0

$$\alpha=-\frac{\int d\tau(-T_2+T_0)}{\hat{S}_0^{(4)}}$$

Quasilinear, Nonlinear, Collisional contributions

$$\frac{\partial f_0(v,t)}{\partial t} = \frac{|e|}{2m} E^*(t) \frac{\partial}{\partial v}\left(f_1(t')e^{-iv(t-t')+\nu(t-t')} + \int_{t'}^{t} ds e^{-iv(t-s)+\nu(t-s)} \frac{|e|}{2m} E^*(s) \frac{\partial}{\partial v}\left(F_0(v,s) + f_0(v,s) + f_2(v,s)\right) + C.C\right) + \nu \frac{\partial^2 f_0}{\partial^2 v}$$

$$\int_0^T dt\sigma^Q = -\int_0^T dt \int dv (F_0 + f_0) \frac{\partial f_0(v,t)}{\partial t} = \left\{\hat{\sigma}_{0,QL}^{(0,2)} + \hat{\sigma}_{0,QL}^{(0,4)} + \hat{\sigma}_{0,QL}^{(2,2)}\right\} + \left\{\hat{\sigma}_{0,NL}^{(0,4)} + \hat{\sigma}_{0,NL}^{(2,2)}\right\} + R_C + R_{Decorr}$$



$$\hat{\sigma}_{0,QL}^{(0,2)} + \hat{\sigma}_{0,QL}^{(0,4)} + \hat{\sigma}_{0,QL}^{(2,2)} = \left(\frac{|e|}{2m}\right)^2 \int dv \left(\int_0^T dt e^{-ivt} E(t) \frac{\partial (F_0 + f_0)}{\partial v}\right)^* \times \left(\int_0^T dt' e^{-ivt'} E(t') \frac{\partial (F_0 + f_0)}{\partial v}\right) > 0$$

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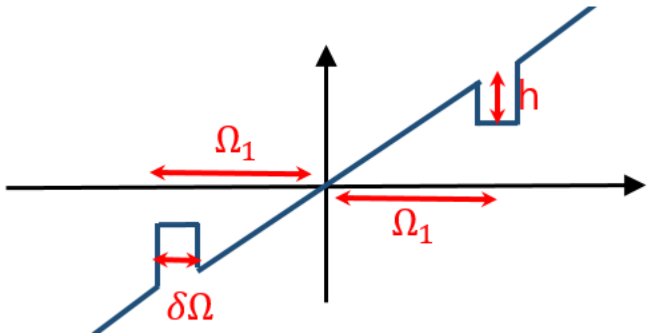
Why the entropy maximization works with nonlinear terms?

- Postulation: Consider a stability of the entropy balance equilibrium

$$\frac{\partial S}{\partial \tau} = \boxed{\sigma_{QL}} + \boxed{\sigma_{NL}}$$



- Long time entropy change is determined by external dissipation

$$\sigma^*_{QL} + \sigma^*_{NL} = D \ll \sigma^*_{QL}$$

- For this chirping case, the quasilinear term is larger than the dissipation term

$$\text{Kubo} = \frac{autocorrelation\ time}{advection\ time} \sim \frac{D}{\sigma_{QL}}$$

$$\sim \frac{1/\nu}{\left(\frac{\partial \Omega_1}{\partial t}\right)^{-1}} \sim \frac{1}{\hat{h}} \frac{1}{\nu} \frac{\delta\Omega^2}{(\gamma_d t)^{1/2}} \ll 1$$

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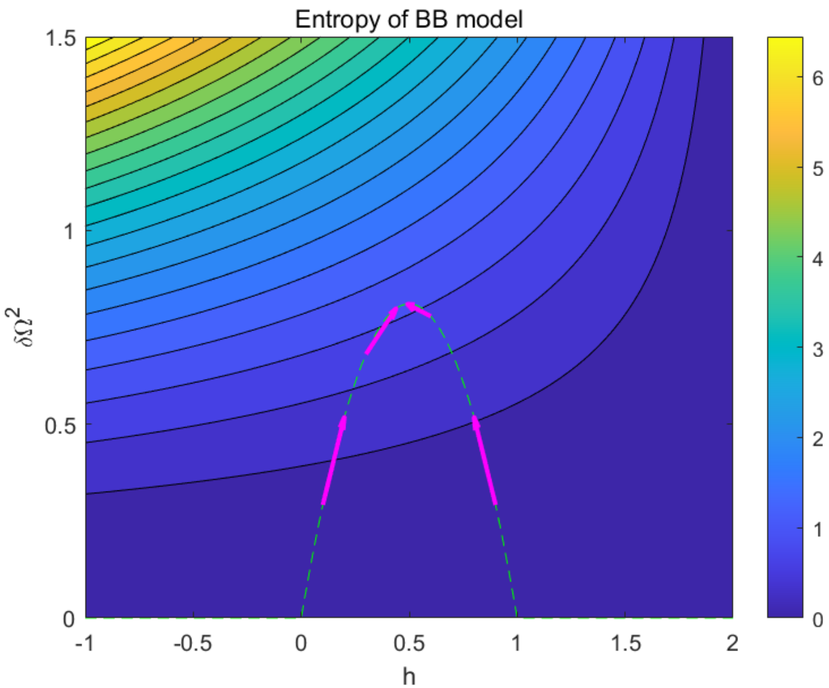
A stability of the equilibrium solution of entropy balance



- The perturbation from the equilibrium determines the parameter change in a short time

$$\frac{\partial(S-S^*)}{\partial\tau} = (\sigma_{QL}-\sigma^*_{QL}) + (\sigma_{NL}-\sigma^*_{NL})$$

Kubo $\ll 1$

$$\frac{\partial(S-S^*)}{\partial h}\frac{\partial h}{\partial\tau} = \left(\frac{\sigma^*_{QL}+\sigma^*_{NL}}{\sigma^*_{QL}}\frac{\partial\sigma^*_{QL}}{\partial h} + \sigma^*_{QL}\frac{\partial(\sigma^*_{NL}/\sigma^*_{QL})}{\partial h}\right)\delta h$$

(1) $\frac{\partial(\sigma^*_{NL}/\sigma^*_{QL})}{\partial h} > 0$ makes unstable equilibrium and increases δh until $\frac{\partial(\sigma^*_{NL}/\sigma^*_{QL})}{\partial h} = 0$

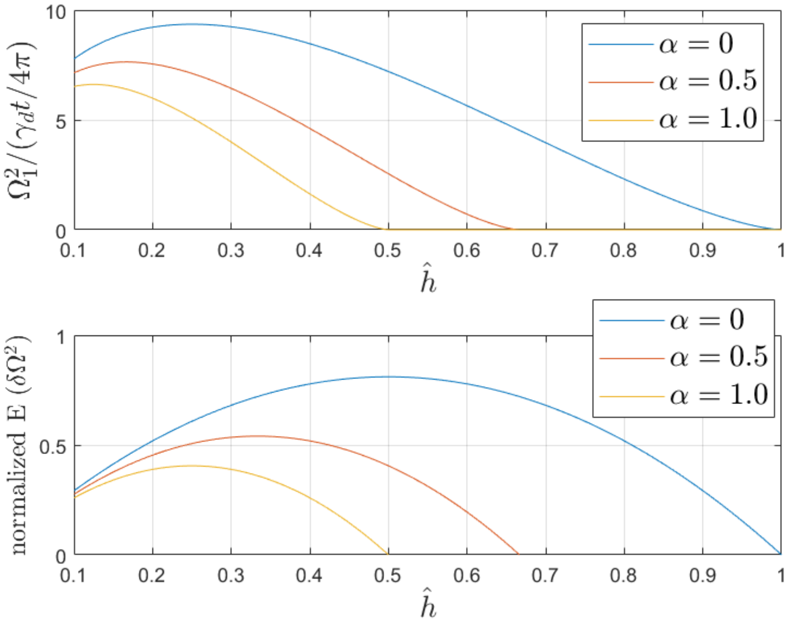
(2) $\frac{\partial(\sigma^*_{NL}/\sigma^*_{QL})}{\partial h} = 0$ could be equivalent to the maximizing entropy with the constraints

The entropy analysis is useful to understand h

- Chirping frequency rate generally increases by the reduced h
→ less time and energy loss to develop h

- Electric fields (trapping width) maximized by a certain h

- Larger α makes the decrease of h, and the increase of the chirping frequency



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Implications of the smaller h

- The smaller h is the increase of the negative energy mode [Fried (1971), Dewar (1973), Lilley (2014)], which is the nonlinearly unstable.

$$\frac{\partial \epsilon}{\partial \omega} \sim 1 - \frac{\Omega_1 + \delta\Omega}{\hat{h}} \left(\frac{(\Omega_1 + \delta\Omega)^2 + \gamma_d^2}{(\Omega_1 + \delta\Omega)^2} \right)^2 < 0$$ can be more likely negative

→ Easy access of the plateau collapse

- Advection (frequency chirping) could be faster for the smaller h.

$$\text{Kubo} = \frac{autocorrelation\ time}{advection\ time} \sim \frac{1/\nu}{\left(\frac{\partial \Omega_1}{\partial t}\right)^{-1}} \sim \frac{1}{\hat{h}} \frac{1}{\nu} \frac{\delta\Omega^2}{(\gamma_d t)^{1/2}}$$

→ in the long time, it becomes quasilinear (Kubo <1). However, a smaller h makes the Kubo number larger (strong turbulence)

$$\Omega_1^2 = \frac{16(\hat{h} - (1+\alpha)\hat{h}^2)}{\hat{h}} \left(\frac{32(\hat{h} - (1+\alpha)\hat{h}^2)}{\pi^2} \right)^{1/2} \frac{\gamma_d}{4\pi} \Delta t$$

$$\delta\Omega^2 = 2\overline{C^2} = \left(\frac{32(\hat{h} - (1+\alpha)\hat{h}^2)}{\pi^2} \right)$$

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Conclusions

- Even in the open, nonlinear, non-equilibrium plasmas, the entropy can be useful to understand the energy transfer and the preferential direction for equipartition.
- In TEM turbulence, the perpendicular diffusion as well as the zonal flow saturation can be described by the entropy calculation
- The energetic particle instability by the BB model can be explained by the free energy balance by defining the effective temperature and heat transfer.
- The entropy change of f0 system shows the strong equipartition, which is likely positive-definite. Maximizing the total entropy with the energy balance and Ampere's law constraints results in the height of the hole and clump, which is useful to understand the BB model in the quasi-adiabatic process.

Thank you for your attention.

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